Curricular content

Number concepts to 1000: understanding place value and the relationship between the digit places and their values

Addition and Subtraction to 1000

Language

Decomposition: breaking a number into its parts. This doesn't always mean by place value.

Compensation:
In addition questions: taking some from one number and giving it to the other in order to make one number easier to work with (usually to the closest ten)

Compensation in subtraction: MUST keep the magnitude of the difference the same. Therefore, if you add to one number in order to bring it to the closest 10 then you must add the same amount to the other number. Likewise, if you subtract from one number to bring it to the closest 10 then you would subtract the same amount from the other number.

Partial sums: when you decompose a number and add parts, then add

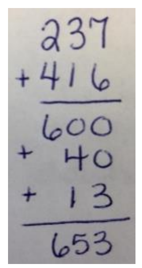
Examples and Strategies

Place value: understanding the value of the digit. Example 836 the "8" is 800, "3" is 30 and "6" is 6

Addition and Subtraction to 100
The key here is to learn the strategies and be able to use them flexibly. Sometimes it will be much easier to use a decomposition strategy, whereas other times the numbers will be easier if you use compensation. Knowing how and when to use the strategies is important.

<u>Addition by Decomposition</u>

$237 + 416 =$



or another example $237 + 416$

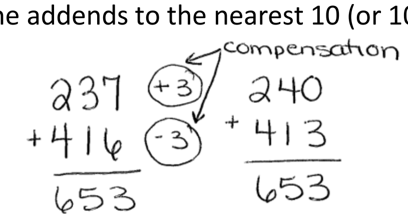
$237 + 400 = 637$
$637 + 10 = 647$
$647 + 6 = 653$

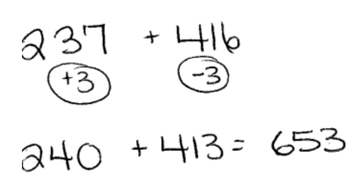
<u>Subtraction by decomposition</u>
Example 672 – 441

$672 - 441 =$

$672 - 400 = 272$
$272 - 40 = 232$
$232 - 1 = 231$

<u>Addition by compensation</u>
Take one of the addends to the nearest 10 (or 100) then compensate by subtracting the equivalent amount from the other addend.
237 + 416 =





The important thing to remember in using compensation to add is that you are keeping the overall quantity the same, and therefore if you increase one addend, you decrease the other.

all the partial sums together to find the total

Difference: finding the magnitude of the difference- or how far apart the numbers are. Very important concept that starts developing early from the sequencing activities where you put numbers in a number line without having to look for each number in order.

Subtraction by compensation

$$\begin{array}{r} 672 \\ -297 \end{array} \quad \begin{array}{c} +3 \\ +3 \end{array}$$

compensate but keep difference the same

$$\begin{array}{r} 675 \\ -300 \\ \hline 375 \end{array}$$

$672 - 297$

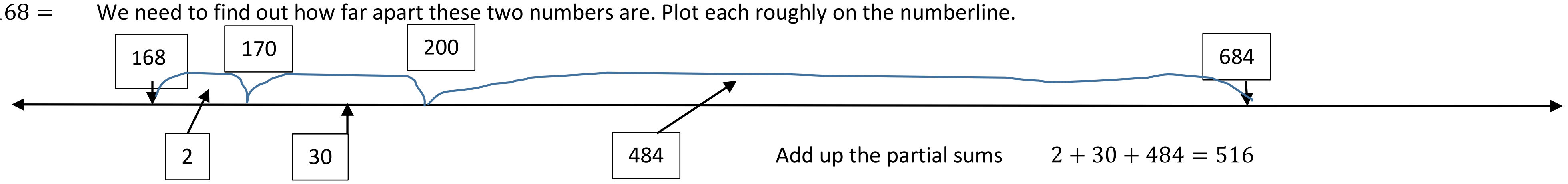
$+3 \quad +3$

$675 - 300 = 375$

The important thing in using compensation with subtraction is that we are keeping the magnitude of the difference the same, therefore if we add to one number, we must add to the other. Likewise, if we subtract from one then we subtract from the other.

Adding up to find the difference

$684 - 168 =$ We need to find out how far apart these two numbers are. Plot each roughly on the numberline.



Add up the partial sums $2 + 30 + 484 = 516$

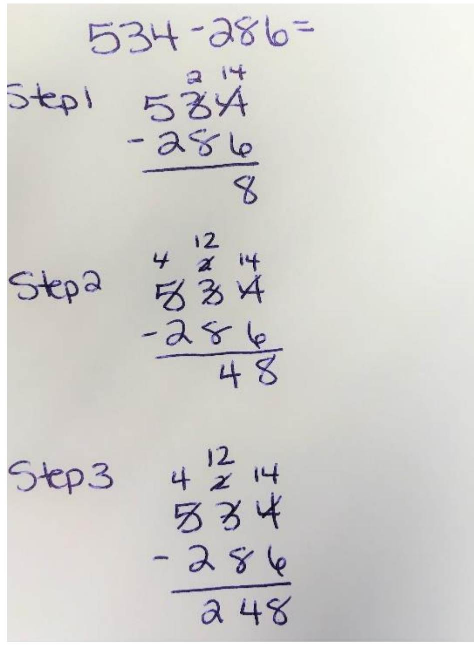
When you add up to find the difference, you can use as many or as few "jumps" as you need to. Some students may be able to see that 168 + 32 gets you to 200, whereas others may need to get to 170 first. *The goal is to be able to do this in two jumps as it is more efficient. For example, be able to find how many to 200 (which is 32) and how far from 200 to 684 (Which is 484) and then add 484 and 32 together to get 516.

***Once students have a very good understanding of adding and subtracting using strategies flexibly, they will need to know how to use the standard algorithm. This is because it is often the most efficient way to perform the required operation. Think of when addition and subtraction become embedded in larger tasks. For example, in division students will need to be able to subtract efficiently in order to divide (whether you are using repeated subtraction for division or other methods). It would quickly become unwieldly to have to subtract by compensation or decompensation at the side of the page, then write in the answer in your division question. For this reason, we teach the algorithm but **only once students have a strong understanding of the concept of addition and subtraction.** It is well worth the time to spend all of grade 2 and a substantial part of grade 3 building that conceptual knowledge and understanding. Do not teach the algorithm in grade 2.

Transitioning to the standard algorithm: Subtraction
It is essential to use manipulatives to transition to the algorithm.
$432 - 267 =$

Step 1: Have three students build the number with base ten blocks and stand at the front of the room. One holds the 4 hundreds flats, one hold three ten rods and one holds 2 ones.
Step 2: Tell the students we are going to subtract 267 from the quantity held at the front of the room.
Step 3: instruct a student to go to the front and take 7 ones. Do not help them. Let them figure out they will need to trade in a ten rod for 10 ones over at the "bank" of base 10 blocks.
Step 4: continue in the same fashion until you have the final solution. Record the answer.

The second time you do a question, do it the same way, but record each transaction on the board.



Step 1: When a ten rod is needed to be traded for 10 ones, record that by showing they now only have 2 ten rods, but now we have 14 ones. Don't have students just sneak a little 1 next to the existing 4. Cross it out and write 14 clearly above the ones column.

Step 2: When a hundreds flat is traded for 10 ten rods, do the same thing and record that you now only have 4 hundreds, but you have 12 ten rods. Again, write it clearly above the tens column.

Step 3: Finish subtracting. The answer is 248.

Step 4: Students should now be able to VERIFY their answer, by adding $248 + 286$ together and they should have 534 altogether. (248 was the difference, 286 was the subtrahend and 534 was the minuend)

Important: Do this MANY times together to make sure everyone understands the concept- not just the procedure. It is important to use manipulatives to develop this understanding. If students are able to flexibly add and subtract smaller numbers using strategies such as decomposition, compensation, adding up to subtract etc, then this is a fairly straight forward leap for students to make. If they don't have that conceptual understanding, this transition to the algorithm will be much more difficult.
Note on terminology: refrain from using "Borrow"- this implies we should "give back" the quantity and that doesn't happen. The overall quantity is the same, we have just changed how it is organized in hundreds, tens, ones.

Where does this lead?

Grade 4 decomposition in multiplication: 26×8

$$(20 + 6) \times 8$$
$$(20 \times 8) + (6 \times 8)$$
$$160 + 48 = 208$$

Grade 9/10: $(2x^2 + 3x - 5) + (7x^2 - 4x + 3) = 9x^2 - x - 2$ (adding polynomials)

Example 2 $2x^2 + 7x + 5$ (Factoring trinomials)

$$2x^2 + 2x + 5x + 5$$

$$2x\,(x + 1) + 5(x + 1)$$

$$(2x + 5)(x + 1)$$

Curricular content

One step addition and subtraction equations with an unknown number

-unknown can be represented by a letter, shape.

Language

Equal: the same as, or balanced

Unequal: not the same as, imbalance

Start unknown: the variable is at the beginning of the statement or equation

Change unknown: a variable is in the middle of the equation.

Result unknown: the final result is the variable which is unknown

Variable: an unknown quantity that we are solving for. Symbols used to indicate the variable might be a box, a letter, a picture etc

Examples and Strategies

Three types of equations:

a) Start unknown example $\blacksquare + 7 = 13 \quad or \;\; x + 7 = 13$

b) Change unknown example $6 + \;\blacksquare = 13$ or $6 + x = 13$

c) Result unknown example $6 + 7 = x$ or $6 + 7 = \;\blacksquare$

It is important to not always have the = in the same place too. You can write it as $13 = 6 + x \quad OR\ 6 + x = 13$. Students sometimes assume that = means “the answer is coming” and we need to ensure they think of = as a balance or “same as”. For example, saying 13 is the same as 6 and 7 rather than always saying 6 and 7 is 13. An example where “the answer is coming next” doesn’t make sense would be $3 + 2 = 4 + 1$ and this is certainly how students will experience equations in more complex math.

Examples: $3 + 4 = 5 + \;\blacksquare$

$\blacksquare - 7 = 10 \quad OR\ x - 7 = 10$ (start unknown)

$17 - x = 10$ (Change unknown)

$17 - \;\; 7 = x$ (result unknown)

Story problem examples:
Start unknown. I had some apples. I ate 7 and now I have 10. How many did I have to start with?
Change unknown: I had $17. I bought some groceries and now I have $10 left. How much did the groceries cost?
Result unknown: I had 17 blocks and Jared took 7 of them. Now how many blocks do I have left?

Where does this lead?
Solve for x
$-3x + 3 = 12$ (grade 8 level) Answer $x = -3$

$5x^2 + 2x - 4 = 3x^2 + 5x - 2$ (grade 11 level)
