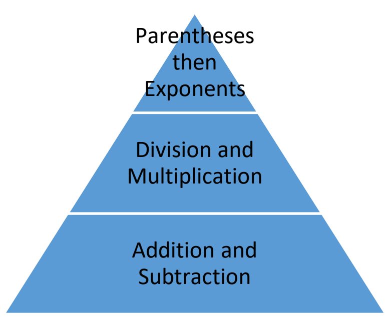
Grade 9 Critical concept: **Operations with rational numbers**

Curricular content

Operations with rational numbers (addition, subtraction, multiplication, division and order of operations)
-incudes brackets and exponents (exponent laws)
-exponents includes variable bases

Language

BEDMAS: or PEDMAS Brackets (or parentheses), Exponents, Division and Multiplication in the order they occur in the equation, Addition and Subtraction in the order they occur in the equation.



Evaluate: determine a value for the expression

Simplify: gather like terms, express in simplest terms

Power: powers have two parts- the base and the exponent.

Examples and Strategies

__**For more discussion of operations with rational numbers refer to Grade 8 critical concept document (fractions) and grade 7 critical concept document (integers)**__

Working with exponents:
Understanding terminology:
2^4
2 is the base
4 is the exponent
2^4 is the power (meaning 2^4 is a power of 2)

__**Important clarification**__: $(-3)^2 \neq -3^2$ When evaluating $(-3)^2$ the base is -3, which means -3 x -3 is 9. In the case of -3^2, the base is 3 and the exponent only applies to the base, not the negative sign. In other words, you do 3 squared first (3x3=9) and then multiply by -1 resulting in -9.

Evaluate: using order of operations

Example #1: $5 + 2 \times (-3)$
$= 5 + (-6)$
$= -1$

Multiply 2 x (-3) then add

Example #2 $4 \div 20 + 1.3$
$= 0.2 + 1.3$
$= 1.5$

Divide 4÷20 first, then add 1.3

Example #3: $14 \div (5 + 2) - 6$
$= 14 \div 7 - 6$
$= 2 - 6$
$= -4$

Add 5 +2 in the bracket first. Then divide 14 by 7 (which was the sum inside the bracket), and finally subtract 6.

Example #4: $8 \times 2 \div 2^2$
$= 16 \div 4$
$= 4$

Exponent first: $2^2 = 4$. Then in order of equation, multiply 8 x 2= 16 then divide by 4

Example #5: $\frac{1}{2} + \frac{3}{5} \div \frac{1}{4}$
$= \frac{1}{2} + \frac{3}{5} \times \frac{4}{1}$
$= \frac{1}{2} + \frac{12}{5}$
$= \frac{5}{10} + \frac{24}{10}$
$= \frac{29}{10} = 2\,\frac{9}{10}$

Divide first, then add. In order to add, you must find the common denominator. Finally, convert from an improper fraction into a mixed number.

__**Exponent laws**__

$x^m \times x^n = x^{m+n}$ example $2^3 \times 2^2 = 2^5$

Think $(2 \times 2 \times 2) \times (2 \times 2)$

$x^m \div x^n = x^{m-n}$ example $2^{5-2} = 2^3$ OR $\frac{2^5}{2^2} = 2^3$

Think $\frac{2\times2\times2\times2\times2}{2\times2}$

Base: The number that is multiplied by itself the number of times indicated by the exponent

Exponent: indicates the number of times the base is multiplied by itself
2^4
2 is the base
4 is the exponent
2^4 is the power

Common denominator: two or more fractions with the same denominator: same size "whole"

Product: value when two or more numbers are multiplied together

Quotient: answer when you divide one number by another
Dividend ÷ divisor= quotient

Sum: answer when two or more numbers are added together

Difference: answer when you subtract one value from another
Minuend- subtrahend= difference

$x^0 = 1$ any base with a zero exponent will have a value of 1.

> The case of the zero exponent! Any base with a zero exponent has a value of 1.
>
> A simplistic way of demonstrating this is as follows: Any number divided by itself is 1. For example $5 \div 5 = 1$ This can be written as $\frac{5}{5} = 1$
>
> We know that $5^1 = 5$ so we can write this equation as $\frac{5^1}{5^1} = 1$ Using exponent laws, this can be written $5^{1-1} = 1$ or $5^0 = 1$
>
> This will be true no matter what base you use (except 0^0 which is undetermined).

$(x^m)^n = x^{mn}$ example $(2^3)^2 = 2^6$

Think $(2 \times 2 \times 2)^2$

$= (2 \times 2 \times 2)(2 \times 2 \times 2)$

$xy^m = x^m y^m$ example $(3 \times 2)^3 = 3^3 \times 2^3 = 27 \times 8 = 216$

Think $(3 \times 2)(3 \times 2)(3 \times 2)$

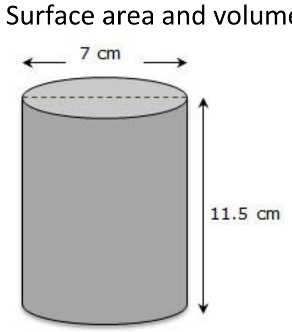
$= 3 \times 3 \times 3 \times 2 \times 2 \times 2$

$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ $y \neq 0$ example $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$

$$\left(\frac{2}{3}\right)^3 = \frac{2 \times 2 \times 2}{3 \times 3 \times 3}$$

Where does this lead?
This is an essential skill for ALL further algebra

Surface area and volume calculations



Radius is equal to half the diameter, therefore:

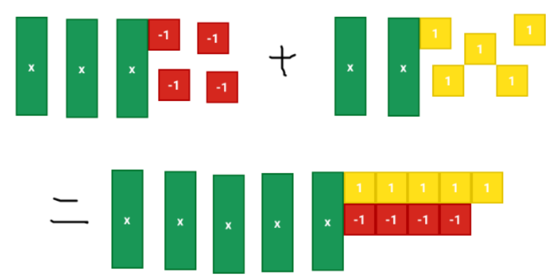
$$r = \frac{1}{2}d = \frac{7\ cm}{2} = 3.5\ cm$$

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi(3.5)^2 + 2\pi(3.5)(11.5)$$
$$SA = 76.969 + 252.898$$
$$SA = 329.9\ cm^2$$

$$V = \pi r^2 h$$
$$V = \pi(3.5)^2(11.5)$$
$$V = 442.6\ cm^3$$

	Exponential functions (Grade 11) The number of students at a particular school who have the flu is increasing at a rate of 15% a day. On Wednesday morning, 50 students have the flu. Approximately how many students have the flu on Friday morning? $A = P(1+r)^t$ $A = 50(1+0.15)^2$ $Approximately\ A = 67\ students$ (Have to round up – can't have part of a student!)

Grade 9 Critical concept: **Polynomials: Operations with polynomials**

Curricular content	Examples and Strategies
Polynomial operations: with polynomials of degree less than or equal to 2 Language **Variables:** a symbol for a number we do not know the value of. Example x and y are variables in the expression 2x – 3y **Degree:** degree of polynomial is the highest degree of the terms. Example $x^2 + 4x - 5$ is a polynomial of degree 2 because the exponent is a 2 in x^2	Adding and Subtracting Polynomials Example: $(3x-4)+(2x+5)$ $=3x-4+2x+5$ Gather like terms $3x+2x+5-4$ $=5x+1$  Common error: Students sometimes don't recognize the difference between a variable and a constant. When using algebra tiles make sure you discuss that the variable does not match up exactly to any whole number of the unit tiles. This is on purpose because the variable can be any value. Before using algebra tiles, try using real life objects such as apples. $(3x-4)+(2x+5)$ can be thought of as "3 apples – 4 + 2 apples +5". You can group together the apples, and group together the constants to have 5 apples +1 but you could NOT group the apples and the constants together. If you are working with equations with two variables e.g. $2x+3y-4+3x-y$ you can think of the x as apples and the y as bananas. You can then group the apples with apples, bananas with bananas, and keep separate from the constants. This helps students see the difference when it becomes more abstract with the algebra tiles. Example #2: Caution around "SIGNS"; e.g. Minus signs between polynomials (between sets of brackets) $(2x^2+3x-4) - (x^2-x-3)$ Watch the signs when you open brackets: It is like multiplying each term in the second bracket by -1 $=2x^2+3x-4-x^2+x+3$ $=x^2+4x-1$

Like terms: term with the same variable as another. The coefficients may be different but the variable is the same

Coefficient: constant number that the variable is multiplied by

Constant: number that has a fixed value

Exponent: the superscript number that shows how many times the base is multiplied by itself
Example: 2^3 means 2x2x2
2 is the base and 3 is the exponent

Simplify: to express in lowest terms: may involve grouping like terms, factoring out common factors,

Distributive: to multiply each term within the bracket by another term
Example: $2x\,(3x+2) = 6x^2 + 4x$

Greatest common factor (GCF): highest number that divides evenly into each of the terms

Polynomial: more than one term

Monomial: consists of one term

Binomial: two terms separated by a + or – sign

Trinomial – three terms separated by a + or – signs

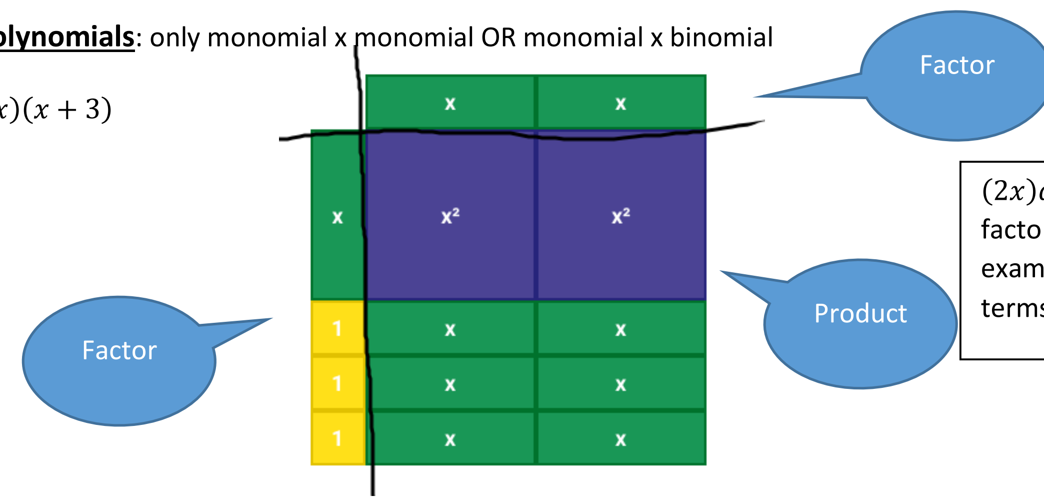
Two variable example:
$(3x^2 - 2y^2 + xy) + (-2xy - 2y^2 - 3x^2)$

Step 1: Brackets
$3x^2 - 2y^2 + xy - 2xy - 2y^2 - 3x^2$

Step 2: Collect like terms
$-4y^2 - xy$

Multiplying polynomials: only monomial x monomial OR monomial x binomial

Example #1 $(2x)(x+3)$
$= 2x^2 + 6x$



$(2x) and\ (x+3)$ are **factors** that we will multiply together to find the **product.** Build the factors and put them at the side and top. Then fill the space in with algebra tiles. The example shows the factors separated from the product by black lines. If you add up the terms in the product you have $2x^2$ and $6x$. Therefore $(2x)(x+3) = 2x^2 + 6x$

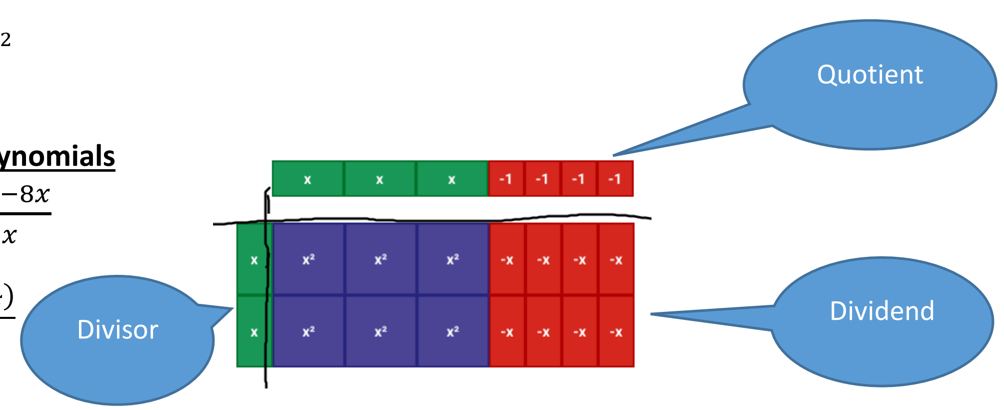
Example #2 using two variables
$(3x)(-2y + 5x)$

$-6xy + 15x^2$

Dividing polynomials

Example: $\frac{6x^2 - 8x}{2x}$

$= \frac{2x\,(3x-4)}{2x}$
$= (3x-4)$



Start with building the dividend in the center, then build the divisor along the side. The quotient will be built on the top side. This diagram shows it separated by a small white space only to make it easier to see that it is the final answer. The quotient can be calculated based on what is needed to match up with the dividend in the center.

	After students are able to understand conceptually using algebra tiles, you can move to simplifying algebraically $\frac{6x^2-8x}{2x}$ can be rewritten as $\frac{6x^2}{2x}-\frac{8x}{2x}$ This can further be simplified to $3x-4$ Example with two variables $\frac{12x^2+6xy}{3x}$ Step 1 write as two fractions $\frac{12x^2}{3x}+\frac{6xy}{3x}$ Step 2: simplify both fractions $4x+2y$
	Where does this lead? Polynomial and rational expressions Grade 11 example $\frac{4\sqrt{5y}}{3\sqrt{2}}$ $=\frac{4\sqrt{5y}}{3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$ $\frac{=4\sqrt{10y}}{3\times 2}$ $\frac{=4\sqrt{10y}}{6}$ $=\frac{2\sqrt{10y}}{3}$ Polynomials and rational equations – Add the following: $\frac{(x^2+3x-5)}{3}+\frac{(2x^2-7x-3)}{4}$ $\frac{4(x^2+3x-5)}{4(3)}+\frac{3(2x^2-7x-3)}{3(4)}$ $\frac{(4x^2+12x-20)}{12}+\frac{(6x^2-21x-9)}{12}$ $\frac{10x^2-9x-29}{12}$

	Inequalities $3(x-6)+2 \leq 5x-4$ $3x-18+2 \leq 5x-4$ $3x-16 \leq 5x-4$ $-2x \leq 12$ $x \geq (-6)$ Factoring and graphing polynomial functions Factor $x^3-3x^2-16x+48$ by grouping. $x^2(x-3)-16(x-3)$ $(x^2-16)(x-3)$ $(x-4)(x+4)(x-3)$

Grade :9 **Critical concept: Equations**

Curricular content	Examples and Strategies
Multi-step one variable linear equations -includes variables on both sides of the equations, rational coefficients, constants and solutions	Example #1 Solve $2x + \frac{1}{10} = \frac{3}{5}$ Step 1) Isolate the variable Subtract $\frac{1}{10}$ from each side (zero pairs) $2x = \frac{6}{10} - \frac{1}{10}$ $2x = \frac{5}{10}$ $2x = \frac{1}{2}$ Step 2) Divide each side by 2 $x = \frac{\frac{1}{2}}{2}$ $x = \frac{1}{2} \times \frac{1}{2}$ $x = \frac{1}{4}$ Check: left side of equation $2\left(\frac{1}{4}\right) + \frac{1}{10}$ $= \frac{2}{4} + \frac{1}{10}$ $= \frac{10}{20} + \frac{2}{20} = \frac{12}{20}$ Left side = right side $\frac{12}{20} = \frac{3}{5}$ Tips for Equations: Don't always use x as a variable- make sure students see other letters as well Include fractions, decimals and integers in the equations Always have students check solutions: use left side must equal right side terminology: equations are always about balance
Language **Variable**: unknown quantity, represented by a letter **Isolate variable:** refers to gathering like terms and then using zero pairs to have the variable on one side of the equation and constants on the other **Like terms**: term with the same variable as another. The coefficients may be different but the variable is the same **Equation**: equality and balance- means that both sides are equal or balanced. An inequality is not an equation. **Expand/distribute:** to multiply each term within the bracket by another term Example: $2x\,(3x+2) = 6x^2 + 4x$	Example #2 $4(a + 1.6) = -3(a - 1.2)$ $4a + 6.4 = -3a + 3.6$ Add $3a$ to each side $7a + 6.4 = 3.6$ $7a = -2.8$ $\frac{7a}{7} = \frac{-2.8}{7}$ $a = -0.4$ Isolate the variable by adding 3a to each side. Then creating zero pairs to isolate variable (results in subtracting 6.4 from each side) Finally, divide each side by 7. Check: left side: $4(-0.4 + 1.6)$ $= 4(1.2)$ =4.8 Right side: $-3(-0.4 - 1.2)$ $= -3(-1.6)$ $= 4.8$

Evaluate: substitute a number for the variable and perform the indicated operations **Solve**: find the value(s) for the variable that satisfy the equation or inequality **Verify**: check your solution to make sure it is true in the original equation	Where does this lead? Solving polynomials and rational equations: Grade 11 example $5+\sqrt{2x-1}=12$ $\sqrt{2x-1}=7$ $(\sqrt{2x-1})^2=7^2$ $2x-1=49$ $2x=50$ $x=25$