Curricular content

2 step equations with integer coefficients, constants, solutions

Expressions: writing and evaluating using substitution

Language

Constant: symbol with a fixed numerical value. (does not change)

Variable: symbol which represents an unknown value e.g. in the equation $3x + 4 = 19$ x represents the variable, 4 is the constant and 3 is the coefficient

Coefficient: number that multiplies the variable e.g. in $4x$, 4 is the coefficient and the variable x is multiplied by 4.

Preservation of equality: understand the equality is a relationship rather than an "operation". This means that what is on each side of the equal sign must be equal. Therefore, we use compensation to keep the quantities equal. For example, if we add a quantity to one side of the equation, we must add the same to the other side. If we multiply,

Examples and Strategies

Expressions: can contain variables, coefficients and constants. An expression does not have a relational symbol such as an equal sign. You can simplify an expression, but not solve it. In other words, you won't be able to determine a numerical value for the variable.

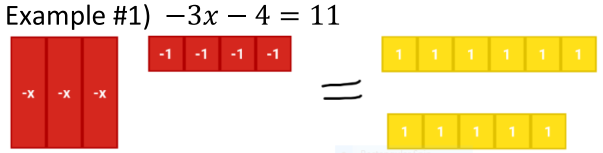
Writing expressions: expressing the relationship. This is extremely important for students to understand and be able to successfully express a given relationship both in words and in expressions.
Example: five less than a number $x - 5$
5 more than three times a number $3x + 5$

Watch for the common error of writing $5 - x$ for "five less than a number"

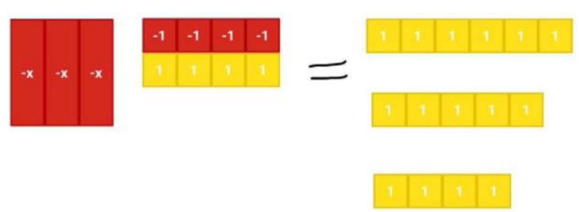
Evaluating an expression using substitution: involves substituting a constant value for a variable example $3x + 5; x = 4$ substitute 4 for x gives you $3(4) + 5 = 17$
Example #2 $0.5x - 2x + 4 \quad if\ x = 8$
$0.5(8) - 2(8) + 4$
$4 - 16 + 4 = -8$

Solving equations
Students will solve 2 step equations with integer coefficients.

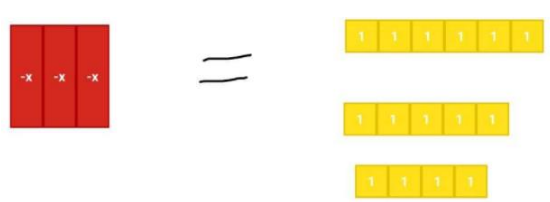
Example #1) $-3x - 4 = 11$



Step 1: build the equation using algebra tiles. Red tiles represent negative values and yellow are positive values.



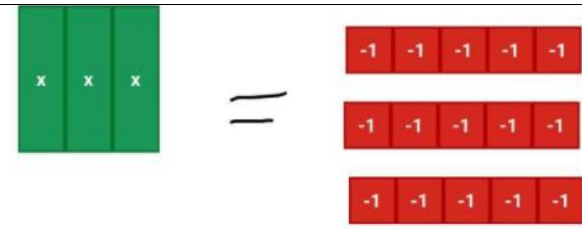
Step 2: Isolate the variable using zero pairs. In order to isolate the x variable on the left side of the equation, we create zero pairs for the constant -4. Since we added +4 to the left side, we must do the same to the right side (resulting in +15 on the right side).



Step 3: shows the variable being isolated on the left side of the equation. In this case, the variable as a negative value. Since we want to know the positive value of x we will need to multiply both sides of the equation by -1.

subtract or divide, the same principle applies.
Use of zero pairs to preserve the equality

Isolate variable: refers to gathering like terms and then using zero pairs to have the variable on one side of the equation and constants on the other



Step 4: we can see that the value of $3x$ is equal to -15. By creating three equal groups on both sides of the equation (dividing both the left and the right side by 3) we get our final answer of $x = -5$

of $x = -5$

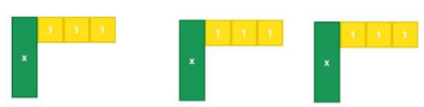
Example #2) $\frac{x}{12} - 6 = 4$

In this case, it gets more difficult to use algebra tiles, although not impossible. It is important for students to understand that $\frac{x}{12}$ means one-twelfth of x
The steps are the same:

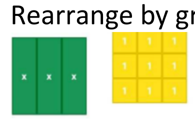
1) Isolate the variable by adding 6 to both the left and the right sides of the equation. $\frac{x}{12} = 10$
2) Now your equation can be read “one twelfth of x is equal to 10.” It is helpful to give a concrete example of what this means. Take a mandarin orange and call it “x" meaning the whole orange is $1\ x$. Break the orange into 12 pieces. $\frac{x}{12} = 10$ means that each of the 12 pieces of the orange is equal to 10. When you put the orange back together, the whole orange is then equal to 120.

You can also do this using a piece of paper. Divide the paper into twelve pieces and put 10 into each section of the paper. The whole paper, considered x, then has 120 counters on it.

Example #3) $-21 = 3(x + 3)$
It is important for students to see the variable on the right side of the equation as well as the left (similar to in primary when we write equations as 13= 7 + □ as well as 7+ □= 13)
Step 1) Use distribution on the right side of the equation. $3(x + 3)$ means “three groups of $(x + 3)$.



Rearrange by grouping the variables together and the constants together



Your equation now reads $-21 = 3x + 9$

Step 2) Isolate the variable using zero pairs
$-30 = 3x$

Step 3)
Divide the left and the right sides of the equation into 3 groups
$x = -10$

Where does this lead?
Grade 9
$4(a + 1.6) = -3(a - 1.2)$
$4a + 6.4 = -3a + 3.6$
Add $3a$ to each side

	$7a + 6.4 = 3.6$ $7a = -2.8$ $a = -0.4$

Grade 8 Critical concept: **Pythagorean theorem**

Curricular content

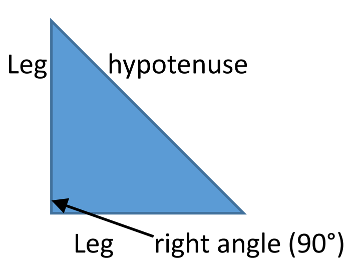
Pythagorean theorem: finding missing side (leg) of a right triangle
-deriving theorem
-includes squares and square roots

Language

Hypotenuse: longest side of a right triangle; opposite the right angle

Legs: other two sides of a right triangle (not the hypotenuse)

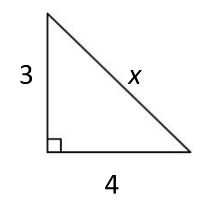
Right angle: 90 degree angle



Examples and Strategies

Finding missing side length of a right triangle: make sure you use a variety of orientations. Don't always have the triangle oriented the way it is in the first example.

Example #1: $a^2 + b^2 = c^2$

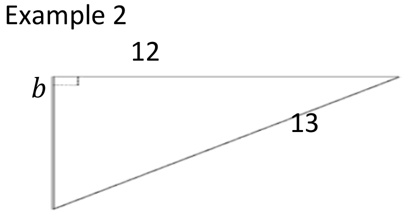


$3^2 + 4^2 = x^2$
$9 + 16 = x^2$
$25 = x^2$
$\sqrt{25} = \sqrt{x^2}$
$x = 5$

Important notes for Pythagorean Theorem

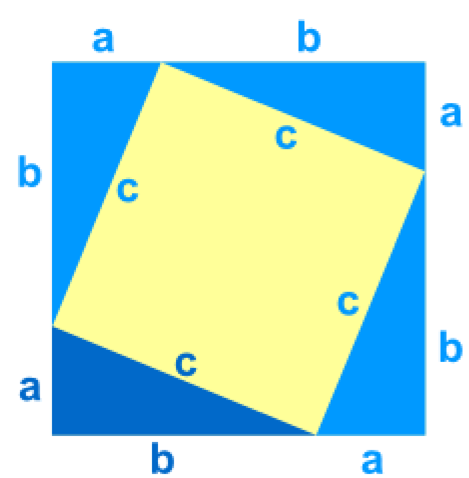
- ✓ Be able to find the <u>hypotenuse and the leg</u>- don't always ask for the hypotenuse
- ✓ Draw the triangle in a variety of orientations

Example 2



$a^2 + b^2 = c^2$
$12^2 + b^2 = 13^2$
$144 + b^2 = 169$
$b^2 = 25$
$\sqrt{b^2} = \sqrt{25}$
$b = 5$

Deriving the theorem



The outer square has side lengths of $a + b$ Therefore the area is $(a + b)(a + b)$ which is $a^2 + 2\,ab + b^2$

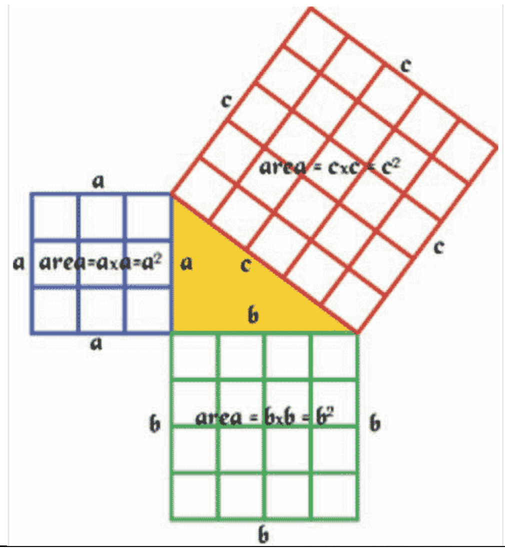
The area of one right triangle is $\frac{ab}{2}$ and since there are four right triangles, the total area is $\frac{4ab}{2}$ or $2ab$

The area of the inner tilted square is c^2

The area of the large square must be equal to the area of the inner square and the four triangles together.

$a^2 + 2ab + b^2 = 2ab + c^2$ which simplifies to $a^2 + b^2 = c^2$

Another example to show the relationship is as below:



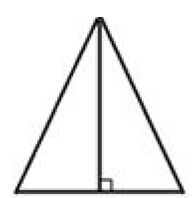
Use graph paper to help students see that the area formed by the square formed on the hypotenuse (side c) is the sum of the areas of the squares formed on the legs (sides a and b) of the right triangle.

In this example, the area of the square formed on the hypotenuse is 25 square units.

The blue square is 9 units² and the green square is 16 units². The sum of these two squares is 25 units²

Where does this lead?

Finding area: This isosceles triangle has two side of length 25 cm and one side length 30 cm. What is the area of the triangle?



To determine the area, you need to find the height of the triangle. $A = \frac{1}{2}bh$

To determine the height of the triangle, we use Pythagorus theorem.

$a^2 + b^2 = c^2$

$25^2 = 15^2 + \; b^2$

$400 = b^2$

$\sqrt{400} = b$

$b = 20\ cm$

Area of triangle $A = \frac{20\times25}{2}$

Area of triangle is 250cm²

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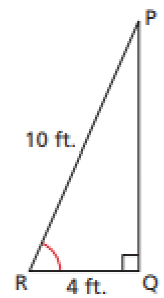
Essential for trigonometry:
Grade 10 example: A 10 ft ladder is leaning against a wall. The base of the ladder is 4 ft from the wall. What angle does the ladder make with the ground?


SOLUTION: using trigonometry
$cosR = \frac{4}{10}$
$cosR = 0.4$
$< R \cong 66°$
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Grade 8

Critical concept: **FRACTIONS**

Curricular content

Operations with fractions: addition, subtraction, multiplication, division

Includes use of brackets but NOT exponents

Language

Simplify (not reduce): Simplifying leaves the expression in the lowest terms. We don't say reduce because that implies changing the size of the pieces (reducing is typically smaller) and we don't want that confusion to arise when we work so hard in fraction work to emphasize SAME SIZES PIECES.

Numerator :how many equal parts we have

Denominator: how many equal parts make up a whole

Fractions are made up of equal size pieces (shares of a whole)

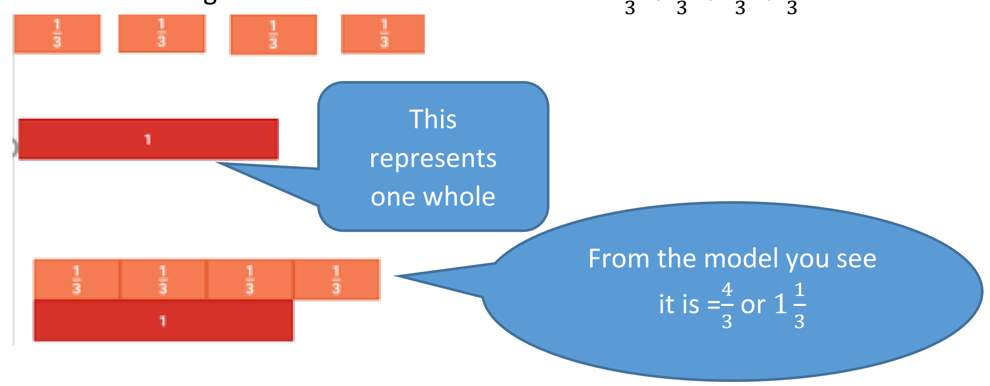
Equivalent fractions: represent the same amount, or same length, but the number of pieces is different

Examples and Strategies

*__important understanding about fractions. Fractions show relationships, not size. One-half is a relationship between the part and the whole. One-half of a large pizza is certainly larger than one-half of a personal size pizza, but the proportion or relationship between the part and the whole is the same.__**

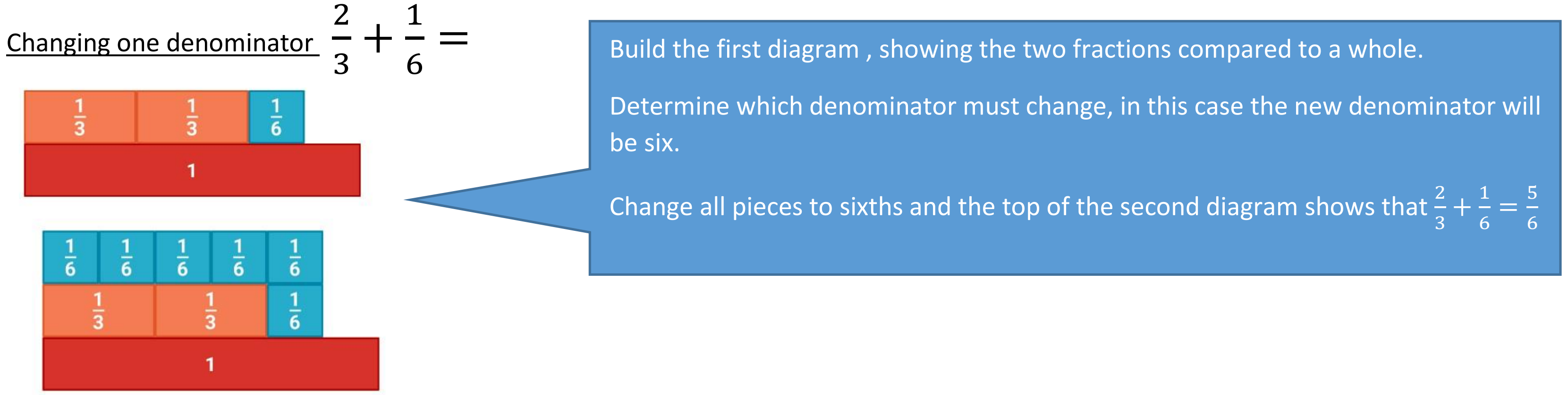
Addition and subtraction of fractions

Start with adding fractions with like denominators. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$



Spend a lot of time adding and subtracting fractions with like denominators before going to unlike denominators. Then move on to where only one denominator needs changing, and finally, to where two or more denominators need changing.

<u>Changing one denominator</u> $\frac{2}{3}+\frac{1}{6}=$



Then changing two or more denominators is the next step.
Since addition and subtraction of fractions follow very similar procedures, the following example illustrates subtraction.

$\frac{2}{3}-\frac{1}{2}=$

Reciprocal: one of a pair of numbers that when multiplied together =1 Every number has a reciprocal except 0. Also called 'multiplicative inverse"

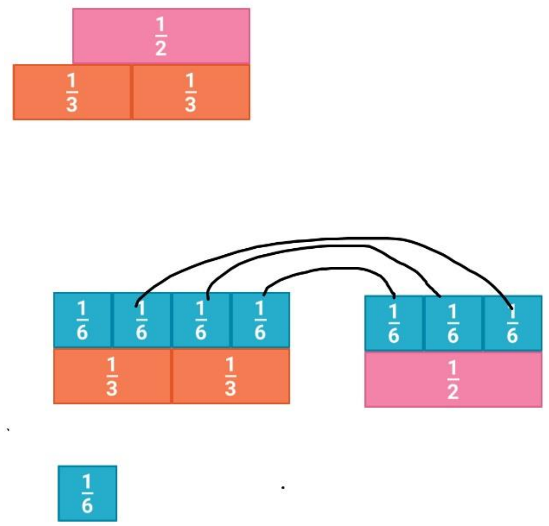
Example: the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ because $\frac{1}{2} \times \frac{2}{1} = 1$

Whole numbers have reciprocals too. The reciprocal of 5 is $\frac{1}{5}$

Commutative property of multiplication: the order in which we multiply the factors will not change the product

Factors: numbers multiplied together to get the product

Product: result of multiplying two or more factors



Remembering this is subtraction…

Build $\frac{2}{3}$ then lay $\frac{1}{2}$ over top. When we subtract we are looking to find the difference between the two. In this case we cannot tell the difference because the pieces are not the same size so we need to compare <u>same size pieces.</u>

The second model shows the equivalent fractions using sixths.

The lines represent removing the three sixths and leaving only one sixth remaining. Therefore the difference is $\frac{1}{6}$

You can do this using hands on manipulatives by laying the pieces directly over top of each other (difficult to show in a diagram)

*Although only one example is shown for subtraction, the progression is the same. Start with same denominators, then only one denominator changing, then two or more.
Do many examples with like denominators before moving on to changing any. This is essential for building the understanding of why a common denominator is needed in addition and subtraction of fractions, but not in multiplication and division.

Common misconception to watch out for: students not understanding that you add or subtract the same size pieces. Many will erroneously believe that $\frac{1}{2} + \frac{1}{3} = \frac{1}{5}$ **<u>BE AWARE!</u>** This misconception is avoided with many examples of adding and subtracting using manipulatives.

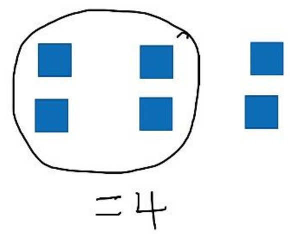
Multiplication and division of fractions

Key understandings:

- ✓ Multiplication can again be expressed as groups of (set model), area model or linear model.
- ✓ Commutative property of multiplication e.g. $3 \times \frac{2}{3}$ is the same as $\frac{2}{3} \times 3$
- ✓ You don't need a common denominator to multiply fractions- because you are **<u>finding part of a part</u>** (therefore the parts don't have to be the same size). In addition you are counting the parts of the whole, so the parts have to be the same size.
- ✓ Multiplication doesn't always result in the product being larger than the factors.

1) Start by multiplying whole number by fraction as in repeated addition $4 \times \frac{1}{3} =$
   This can be read 4 groups of $\frac{1}{3}$ and thought of as $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$

2) You can multiply a fraction by a whole number such as $\frac{2}{3} \times 6 =$ Partition 6 into three groups, then select 2 of those three groups.



Partitioning 6 into 3 groups, and selecting two of those groups is essentially $6 \div 3 \times 2$ which easily transitions into division later on.

3) Multiplying fractions by other fractions

$$\frac{3}{5} \times \frac{3}{4} =$$

Step #1

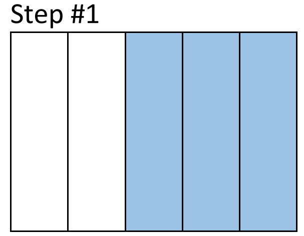
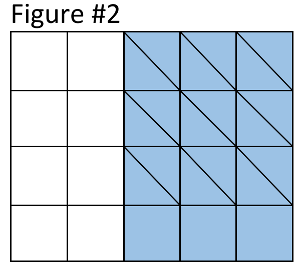


Figure #2



Step #1 Shade in 3 of the 5 columns

Step #2 Shade in $\frac{3}{4}$ of the shaded region. Since you can't easily do that, you must partition the entire table into four equal pieces and then shade in three of the four. See Figure 2

The slashed boxes represent $\frac{3}{4}$ of the shaded region, which is 9 of the 20 boxes, or $\frac{9}{20}$

Therefore,

$$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$$

This means we have 9 of the pieces needed to make one whole, which has 20 pieces.

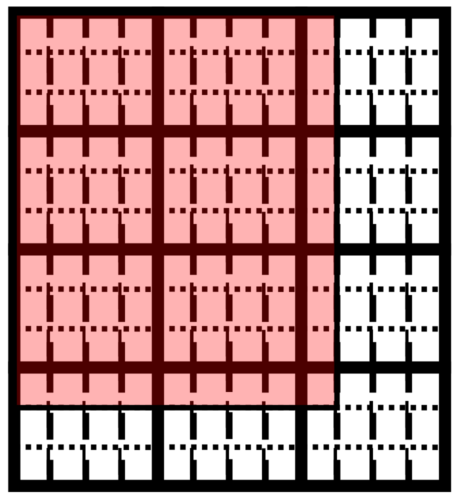
*NOTE the <u>area of the whole stays the same</u>→we are only finding <u>part of the part</u>

The area model (shown above) is the easiest way to visually see multiplying fractions, and also easily transitions into the standard algorithm. However, if we simply teach the method of "multiply top and bottom" students easily misunderstand what the product actually represents. **It is highly recommended to use the area model for many days and by then the students will quickly see the transition to algorithm for speed purposes. Build understanding first.**

Multiplying mixed numbers

Example $2\frac{1}{4} \times 3\frac{1}{3}$

Using the area model: this follows exactly the same strategy as multiplying whole numbers.



Remember, the side lengths are the factors in the area model of multiplication. Therefore, the top side length is $2\frac{1}{4}$. You can see that there are 2 complete grids across the top shaded in, and one of the four strips that make up the third grid.

The other side length shows three and one third grids shaded.

The area formed by the rectangle created is the product of $2\frac{1}{4} \times 3\frac{1}{3}$

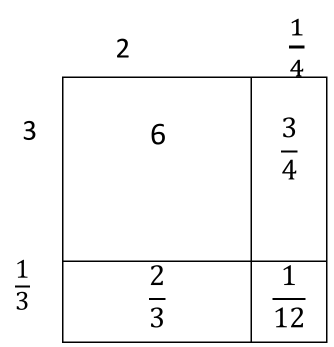
Because we had to divide our grids into four columns and three rows, one whole now has 12 small boxes.

Our model shows 9 shaded squares x 10 shaded squares for a total of 90 shaded squares. (1 whole=12 squares)

$\frac{90}{12} = 7\frac{6}{12}$ which simplifies to $7\frac{1}{2}$

Using a sketch of the area model

$2\frac{1}{4} \times 3\frac{1}{3}$



Adding up all the partial products:

$6 + \frac{3}{4} + \frac{2}{3} + \frac{1}{12}$

$= 6 + \frac{9}{12} + \frac{8}{12} + \frac{1}{12}$

$=6 + \frac{18}{12}$

$=6 + 1\frac{6}{12}$

$=7\frac{1}{2}$

Transition to the standard algorithm:

$2\frac{1}{4} \times 3\frac{1}{3}$

Remember we are finding part of a part, so we don't need a common denominator.

$2\frac{1}{4} \times 3\frac{1}{3}$

$= \frac{9}{4} \times \frac{10}{3}$

Connect this to the area model shown above. We had 9 squares along the top (columns) and 10 rows=90 squares. The size of the whole was 12.

Numerator: 90 (number of squares we have)

Denominator=12 (size of the whole)

$=\frac{90}{12}$ which simplifies to $7\frac{1}{2}$

**Be very specific about showing the connection to the model when you are developing the algorithm for multiplying fractions. Using the algorithm is fast (and generally easy) BUT you need to understand what the meaning is.

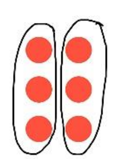
Division with fractions

Key understandings

- ✓ Division does not always result in a smaller quotient
- ✓ Vocabulary is important! Dividing BY $\frac{1}{2}$ different from dividing IN $\frac{1}{2}$. Always go back to estimating "should the result be larger or smaller than the original amount?"
- ✓ When using the algorithm (after strong understanding is built) we multiply by the reciprocal (NOT invert and multiply)

Division can be thought of as sharing equally.

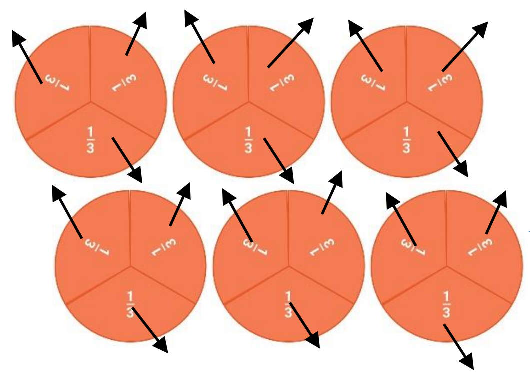
$6 \div 3 = 2$ because if you share 6 into groups of 3 you will have 2 groups



Example with fractions

$6 \div \frac{1}{3} =$

Means sharing 6 whole into groups of $\frac{1}{3}$



In this example, we are sharing 6 into groups of $\frac{1}{3}$ which results in 18 groups (and each group has $\frac{1}{3}$ in it)

Key understanding is to see clearly that dividing by $\frac{1}{3}$ results in three times as many groups. This makes the transition to the standard algorithm make sense.

Real life illustration of the above example:

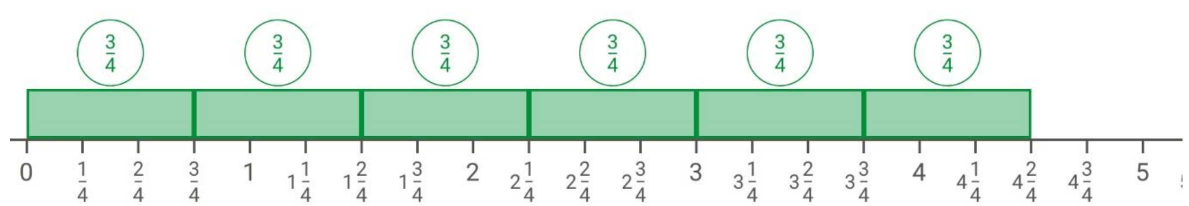
You have 6 large cookies.

a) How many people can you feed if each person gets 3 cookies? (top model-You can feed two people)

b) How many people can you feed if each person gets $\frac{1}{3}$ cookie? (second model- you would feed 18 people)

Example #2

You are making raspberry jam. Each batch of jam requires $\frac{3}{4}$cups of sugar. You have $4\frac{1}{2}$ cups of sugar. How many batches will you be able to make? Answer: 6 batches



First- decide if the answer will be larger or smaller than $4\frac{1}{2}$.

Using manipulatives, number lines etc find how many groups of $\frac{3}{4}$ can be made from $4\frac{1}{2}$. From the model to the left, you can see you will make 6 groups. Therefore

$$4\frac{1}{2} \div \frac{3}{4} = 6$$

Transitioning to the standard algorithm

Use the first example about the cookies. Students will quickly see that if you divide into serving sizes of $\frac{1}{2}$, you will be able to feed twice as many people as you would if they each got a whole cookie.

$6 \div \frac{1}{2} = 12$ (meaning 12 people would get half a cookie each)

	$6 \div \frac{1}{3} = 18$ $6 \div \frac{1}{4} = 24$ Students will start to see the pattern that when you divide by $\frac{1}{3}$ you get 3 times as many. When you divide by $\frac{1}{4}$ you get 4 times as many and so on. This is the equivalent of multiplying by the **reciprocal. (or multiplicative inverse)** Therefore, you can transition to the algorithm $6 \div \frac{1}{4} =$ $6 \times \frac{4}{1} = 24$ *Note* when dividing by a fraction we can multiply by the reciprocal (not 'invert and multiply')
	Where does this lead? Grade 11: Dividing rational expressions $\frac{x^2-4}{x^2-4x} \div \frac{x^2+x-6}{x^2+x-20}$ $=\frac{(x-2)(x+2)}{x(x-4)} \div \frac{(x+3)(x-2)}{(x+5)(x-4)}$ $=\frac{(x-2)(x+2)}{x(x-4)} \times \frac{(x+5)(x-4)}{(x+3)(x-2)}$ Simplify and multiply $=\frac{(x+2)(x+5)}{(x)(x+3)} \quad x \neq -5, -3, 0, 2, 4$