Grade 6

Critical concept: **Factors and Multiples**

## Curricular content

Factors and Multiples: includes prime factorization, factor trees, divisibility rules

Greatest common factor

Least common multiple

## Language

Prime number: positive integer other than 1 that can only be divided evenly by itself and 1

Composite number: positive integer other than one that has at least one positive divisor other than one or itself

Venn Diagrams: used to classify numbers as prime, composite, also for GCF and LCM

Divisibility Rules: way of testing if a number is evenly divisible by a divisor

LCM: least common multiple: loweset positive integer that is evenly divisible by all numbers in a set

GCF: greatest common factor

## Examples and Strategies

It is helpful to understand and know the divisibility rules before working with GCF and LCM.
Divisibility rules are easy ways to check if a number is evenly divisible by a divisor

RULES

| Divisible by | If... |
|---|---|
| 2 | It is an even number ending in 0,2,4,6,8 |
| 3 | The sum of the digits is a multiple of 3 |
| 4 | Last two digits are multiples of 4, or are 00 |
| 5 | The number ends in 0 or 5 |
| 6 | It is divisible and 2 and 3 (even and divisible by 3) |
| 7 | Double the last digit and subtract it from the number made by the other digits. If the answer is divisible by 7 then the number is divisible by 7 |
| 8 | Last three digits are divisible by 8 |
| 9 | Sum of digits is multiple of 9 |

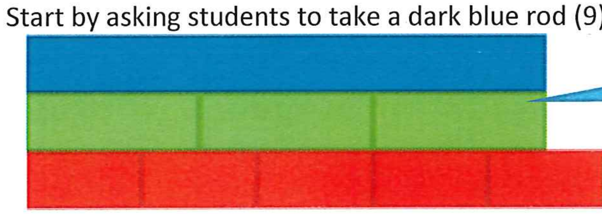
It is critical that students understand the difference between **factors and multiples**. If students have learned the area model of multiplication, it is important to make the connection that factors are the side lengths of the array. The side lengths of the array or rectangle are the ***factors.*** If you cannot make a rectangle with a side length of "3" for example, then 3 is not a factor of that particular number.

In this example the rectangle has a length of 5 and a width of 3.
The factors are 5 and 3 and the product (area covered) is 15

**FACTORS**

Factors: students must understand the connection to multiplication and the area model.
If you picture the rectangle or array, the length of each side is a factor. This will be absolutely critical to understand when you are factoring trinomials in Grade 10. Being able to find the greatest common factor will be equally as important in factoring and is well worth the time spent in grade 6 learning to find GCF's.

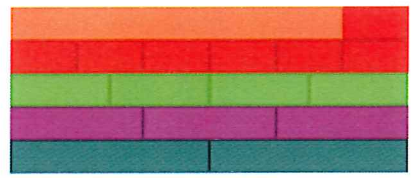
We can also use cuisennaire rods to illustrate factors.
Start by asking students to take a dark blue rod (9). Make this length using other colour rods (all the same colour)



You can make dark blue length using light green because 3 is a factor of 9 (and 9 is a multiple of 3)

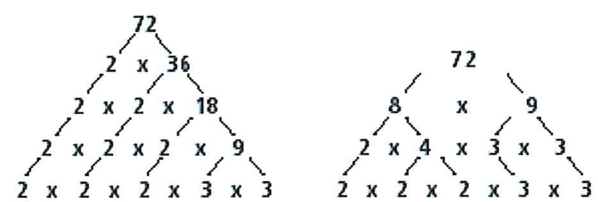
You cannot make dark blue length using red because 2 is not a factor of 9 (and 9 is not a multiple of 2)

Finding the Greatest Common Factor between 8 and 12



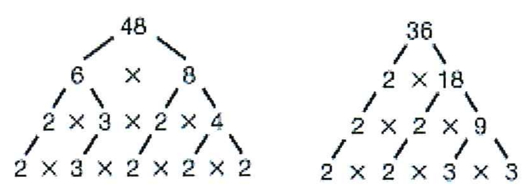
Build factor trains for each of 12 and 8. From this, you can see that the common factors are 2 (red) and 4 (purple). The greatest of these two common factors is 4; therefore the GCF of 12 and 8 is 4

Prime factorization: factor trees




The two examples to the left show two methods of arriving at the same prime factors for 72. Being able to use prime factorization will help students identify quickly the GCF for pairs of numbers that are much too large to be building with manipulatives.

<u>Using Prime Factorization to find GCF of large numbers</u>



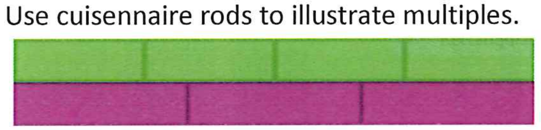

Common factors: 2,2, 3

GCF is 2x2x3=12

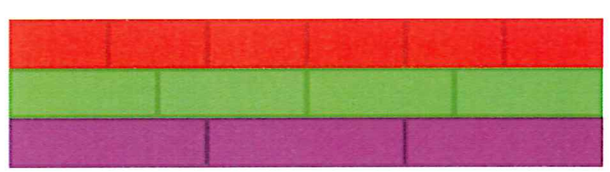
**<u>MULTIPLES</u>**

Multiples are numbers that can be divided evenly by a given number. For example multiples of 3 are 3,6,9,12… Make the connection with students that if they start skip counting at the particular number then they are usually counting multiples. E.g. skip counting by 4's starting at 4 is the same as saying the multiples of 4.

Working with multiples:
Use cuisennaire rods to illustrate multiples.



In the first example, we line up multiples of 3 and multiples of 4 to find the first place that the factor trains come to the same length. You will see that 12 is the least (lowest) common multiple of 3 and 4.



In this example, we are finding the LCM of 2,3, and 4. Build factor trains of each factor and see where the first time is where the trains come to the same length. In this case, the least common multiple is 12.

Finding the LCM is going to be very important when working with fractions→ <u>common denominators!</u>

**Where does this lead?**

Fraction operations – grade 8

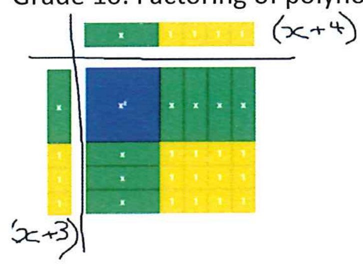
Adding fractions $\frac{1}{4} + \frac{3}{2} + \frac{2}{5} =$

$\frac{5}{20} + \frac{30}{20} + \frac{8}{20} =$

$= \frac{43}{20}$

$= 2\frac{3}{20}$

Grade 10: Factoring of polynomials using the area model!! Factor $x^2 + 7x + 12$ (Build your rectangle and the side lengths are the factors)




Rational expressions Grade 11

Simplifying radicals :

$\sqrt{50}$

$=\sqrt{25}\,\sqrt{2}$

$=5\sqrt{2}$

**Grade 6** Critical Concepts: **Ratios**

**Curricular content**

Introduction to Ratios
Equivalent Ratios

Part : Part ratios

Part : Whole ratios

**Language**

Ratio: relationship between two or more quantities. Ratios have the same unit. (Rates have different units e.g. km/hr)

Ratios are proportions. E.g. for every 4 reds there is 1 blue. Or for every 2 cups of water we use one cup of orange juice concentrate.

Part to part ratios are NOT fractions. Do not express them as fractions. Example the class has 11 girls and 8 boys. The ratio of girls to boys is 11:8 but this is NOT a fraction of $\frac{11}{8}$.

Part to whole ratios can be expressed as fraction. The above example of 11 girls and 8 boys can be a part to whole ratio of girls to students in class of 11:19 which CAN be expressed as a fraction $\frac{11}{19}$

Equivalent ratios: keep the <u>proportion</u> the same.

Terms of the ratio: the individual portions are the "terms" of the ratio. For example in the ratio of girls to

**Examples and Strategies**

All fractions are ratios, but all ratios are <u>not</u> fractions. This is an important distinction. The fraction $\frac{3}{7}$ means 3 out of the 7 pieces that make the whole. A story for this may read "Three of the seven children are girls". As a ratio you could say 3:7 This is a **part to whole ratio**, meaning 3 of the 7 children are girls, and this is a fraction. However, if you express this as a **part: part** ratio by saying 3:4 there are 3 girls to every 4 boys, then you cannot express this as a fraction. (Writing that ratio as a fraction would be saying there are 3 girls out of the 4 boys, which is incorrect)

You can also have more than two terms in a ratio but not a fraction. Example 3:2:4 could be a ratio of lemonade: orange juice: sprite in a punch. (for every 3 cups of lemonade there would be 2 cups of orange juice and 4 cups of sprite). Notice that when writing ratios order matters!

*It is critical that students understand what is being represented by the ratio and can explain and recognize the difference between part to part and part to whole ratios.*

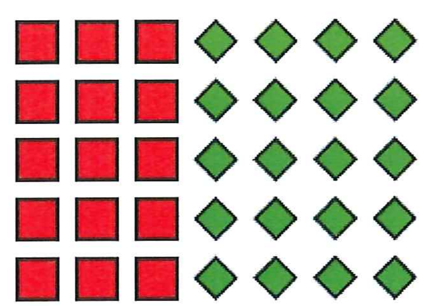
Part to Part and Part to Whole ratios

**Part to part ratio** is the ratio of green diamonds to red squares 4:3 or 20:15
**Part to whole** ratio could be the ratio of green diamonds to all shapes 4:7 or 20:35

ORDER MATTERS!

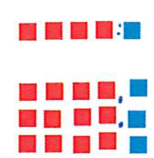
The order of the ratio is important. If you ask for the ratio of diamonds to squares you must put the diamonds first in the ratio e.g. 4:3

If you wrote 3:4 you would be describing the ratio of red squares to green diamonds.



**Equivalent ratios (make the connection to equivalent fractions here as well)**

1) Coloured tiles
4:1
12:3



In the original set there were four reds for every one blue

When we created 3 sets of 4 reds, we created 3 sets of one blue

Red tiles to blue tiles is a part to part ratio.

A part to whole ratio would be a ratio of red tiles to all tiles. In this case it would be 4:5

| | |
|---|---|
| boys 11:8 the numbers 11 and 8 are the terms of the ratio. | Real life examples :<br><br>Making Orange Juice for every can of OJ concentrate, 3 cans water concentrate: juice. If you need to make 6 cans of OJ how much water do you need?<br><br>1 OJ : 3 water<br>6 OJ: 18 water<br><br>Planting 12 acres of corn takes 3 hours . How long will it take to plant 16 acres?<br><br>12 acres: 3 hours<br>16 acres: n hours |
| | Where does this lead?<br><br>Similar triangles: Triangle ABC is similar to triangle XYZ. If side AB is 10cm, side BC is 15 cm and XY is 2 cm, how long is side YZ?<br>Solution<br>$\frac{AB}{BC} = \frac{XY}{YZ}$<br><br>$\frac{10}{15} = \frac{2}{YZ}$<br><br>$YZ = \frac{2 \times 15}{10}$<br><br>$= \frac{30}{10}$<br>=3 cm |

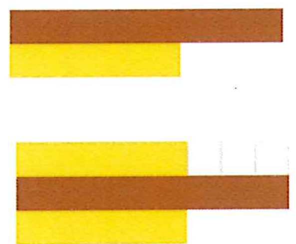
Curricular content

Improper fractions and mixed numbers
(converting between improper fractions and mixed numbers; comparing using numberlines and benchmarks)

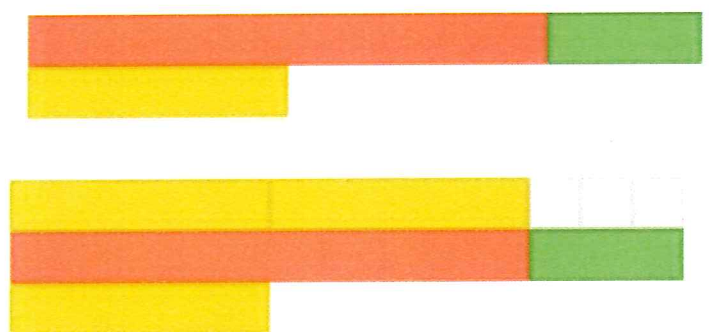
Examples and Strategies

In grade 6 students should already have a good understanding of equivalent fractions (from grade 5). The new understanding in grade 6 is that fractions can represent more than one whole. <u>Fractions do not always mean less than 1. This is a common trouble spot for students.</u>

You can build this understanding using Cuisenaire rods.



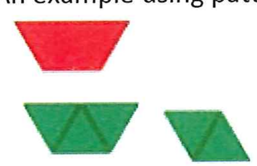
The yellow rod represents the **<u>whole.</u>** The fraction shown represents having more than one whole, as the numerator (brown) is greater than the whole (yellow). We can show this with a mixed number $1\frac{3}{5}$ which is shown in the second illustration- the brown rod is the same as one yellow rod (the whole) plus three more unit cubes (+3). This is one whole, and three of the five pieces needed to make another whole.



The top illustration shows $\frac{13}{5}$ (13 is shown as 10 +3). This is an improper fraction. Converting to a mixed number we need to find out how many "Wholes" we can make from 13 given that the size of a whole is 5.

The second part of the diagram shows we can make 2 whole yellow rods, and 3 of the 5 pieces needed for the next, which can be written $2\frac{3}{5}$

An example using pattern blocks

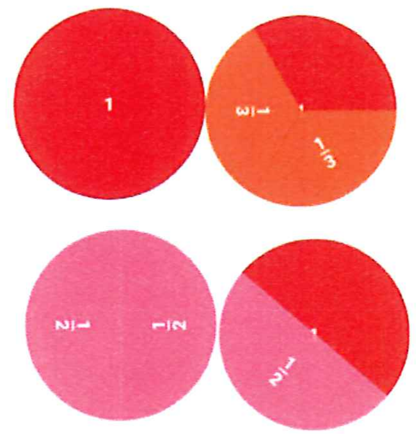


If the red trapezoid is considered one whole, then there are three green equilateral triangles that make up one whole. Since we have 5 green triangles, we can say we have $1\frac{2}{3}$ because the two green triangles make up $\frac{2}{3}$ of the red trapezoid. This can be converted to an improper fraction. We have $\frac{5}{3}$, meaning we have 5 pieces and it takes 3 pieces to make up a whole.

Comparing fractions and mixed numbers

Again, it is nice to use manipulatives to compare fractions and mixed numbers in order to build conceptual understanding.

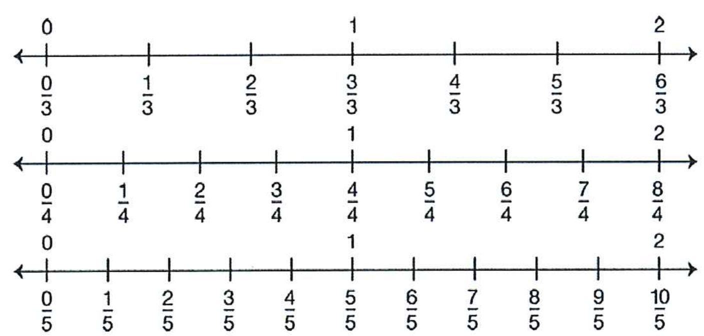
Example #1 Which represents the larger amount $1\frac{2}{3}$ OR $\frac{3}{2}$?




Fraction circles are useful for comparing fractions as well. In a fraction circle you can compare without finding the common denominator because the whole is the same size. Here you see one whole and two third compared to the second set showing $\frac{3}{2}$

Students need to be able to compare the size of fractions and mixed numbers- for example, by placing them on the number line.
Example: compare and order the following fractions by placing them on a numberline

$\frac{5}{4}$, $\frac{2}{3}$, $1\frac{1}{3}$, $\frac{3}{4}$, $1\frac{2}{5}$




This set of number lines allows us to compare thirds, quarters and fifths. Note that the numberline starts at 0 and is marked to 2 and the fractions are expressed as improper fractions rather than mixed numbers. Students should be comfortable with both forms.

By converting the mixed numbers to improper fractions, students will see that $1\frac{1}{3} = \frac{4}{3}$ and $1\frac{2}{5} = \frac{7}{5}$ and then the fractions can be easily ordered.

Common errors to watch for:

- Uneven divisions to mark the fractions
- Not understanding a whole, or two wholes (e.g. placing $\frac{5}{5}$ to the left of the 1 rather than understanding they are equivalent)

Language

Numerator: how many parts we have

Denominator: how many equal parts make up a whole- the size of the whole is very important when comparing fractions

Where does this lead?
Operations with fractions: Grade 8 example

$4\frac{2}{5} \div 1\frac{1}{2} =$

SOLUTION

$\frac{22}{5} \div \frac{3}{2} =$

$\frac{22}{5} \times \frac{2}{3} = \frac{44}{15}$

| | |
|---|---|
| Fractions are made up of equal size pieces (shares of a whole)<br><br>Common Denominator: making the number of pieces in the whole the same when working with two or more fractions<br><br>Improper fraction: the numerator is greater than the denominator; there is more than one whole represented by the fraction<br><br>Mixed numbers: consists of a whole number and a fraction e.g. $2\frac{3}{4}$<br><br>*<u>Simplify</u> don't reduce (you are not changing the size) | $=2\frac{14}{15}$<br><br>Rational expressions: Grade 11 example<br>$\left(\frac{4}{2x+6}\right)+\left(\frac{5}{x+3}\right)=$<br><br>SOLUTION<br>$\frac{4}{2(x+3)}+\frac{5}{x+3}$<br><br>$=\frac{4}{2(x+3)}+\frac{10}{2(x+3)}$<br><br>$=\frac{14}{2(x+3)}=\frac{7}{x+3}$<br><br>$x \neq (-3)$ |

Grade 6 Big Idea: **Percent**

Curricular content

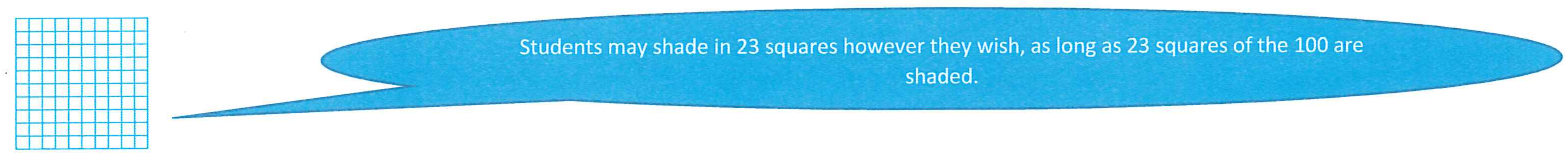
Whole number % and percentage discounts

Finding missing part

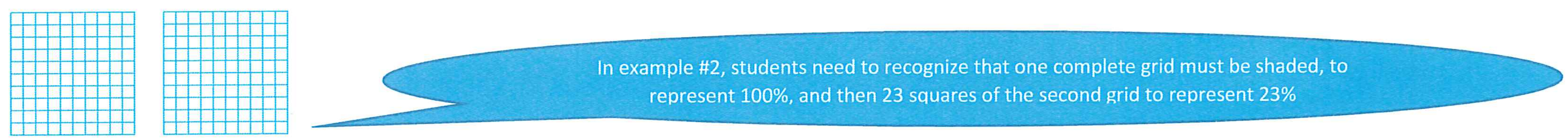
Examples and Strategies

Students need to clearly understand that percent is simply a fraction with a denominator of 100. 100% means $\frac{100}{100}$ which is 1 whole. Less than 100% means less than 1 whole. Greater than 100% means more than one whole. A common misconception is students not understanding that you can have more than 100%.

Example #1
Shade in 23%




Example #2
Shade in 123%




Percentages students should learn to recognize immediately:
100% (one whole)
50 % = one half
25%= one quarter
10% is easily calculated by dividing by 10. Make sure students understand why, and then practice mental math calculations using 10% as a bench mark to calculate percentages such as 30% easily

Example:
Find 30% of 80
Mentally calculate 10% of 80 is 8
30% of 80 is then $8 \times 3 = 24$

| | |
|---|---|
| | **Percentage discounts**<br><br>Finding percentage discounts is similar to the examples shown above. Once the discount is calculated, it must be subtracted from the original price in order to determine the new price. This is a common oversight.<br><br>Example: Your favourite clothing store is having a sale on jeans. The regular price of the jeans is $84 and they are now on sale for 20% off. What is the new price, before taxes?<br><br>Steps:<br>1) Calculate 10% of $84 which is $8.40.<br>2) 20% will then be $2 \times \$8.40 = \$16.80$<br>3) Since $16.80 is the discount, we then need to subtract this from the original purchase price of $84<br>$\$84 - \$16.80 = \$67.20$<br>Therefore, the final price, before taxes, is $67.20 |
| Language<br>Percent = out of 100<br><br>Discount: amount that is subtracted from the original price | Where does this lead?<br>Sales tax calculation: Grade 7 question<br>You are buying a pair of shoes that are on sale for 20% off. The regular price is $84. What is the sale price? What is the final total including sales tax of $12%<br><br>Find 20% of $84<br>$0.2 \times \$84 = \$16.80$<br>The discount is $16.80 which must then be subtracted from the original price.<br>$\$84 - \$16.80 = \$67.20$<br><br>Sales tax is 12% so you must now calculate the tax.<br>$0.12 \times \$67.20 = \$8.06$<br>Add tax to the price<br>$\$8.06 + \$67.20 = \$75.26$<br><br>Or you could use the strategy of knowing that you pay 1.12 times the sale price<br>$\$67.20 \times 1.12 = \$75.26$ |