

Curricular content
Multiplication of 2 or 3 digit numbers by one digit numbers
Language
Array: a rectangular representation of multiplication of two numbers. For example: a rectangular array made of 4 rows of 6 tiles (24 tiles in total)
Factor: a number that you multiply with another number to get a product
Product: the result when two or more factors are multiplied together.
Distributive property: partitioning a number by place value and multiplying each addend separately by the factor and adding the products together. (partial products)
Commutative property: order doesn't matter in multiplication
Area model: creating a rectangle with each side length being one of the factors

Examples and Strategies

In grade 3 we start multiplication by building many arrays. If students haven't done this yet, start there. Put 18 tiles down. Have students build arrays. The number of tiles in each row and column are FACTORS of the number 18. You can't make a rectangle with side length of 5 using 18 tiles because 5 is NOT a factor of 18.

The next steps to move ahead are:

- Multiplying by multiples of 10:
 $3 \times 1 = 3$
 $3 \times 10 = 30$
 $3 \times 100 = 300$

Show these using base ten blocks

- Multiplication by decomposition – this is an important step in understanding multiplying multiples of 10- once they have seen it and understood it, go to step 3.

Example $20 \times 30 =$
 $(2 \times 10) \times (3 \times 10)$
 $(2 \times 3) \times (10 \times 10)$
 $6 \times 100 = 600$

20 is 2x10 and 30 is 3x10. Commutative property of multiplication shows we can change the order of the factors and associative property allows us to group them in a way that makes it easier. Make it easier by grouping 2x3 and 10x10.

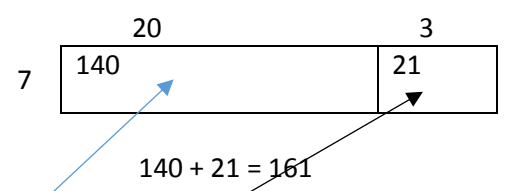
It is important to understand this, rather than simply saying multiply the 2 and 3 and put 2 zeros on the end. (see next step to explain why)

- Annex zero **(please don't say ADD zeros- when we add zero to a number the quantity doesn't change- and this will be important when we do algebra and zero pairs)**

$20 \times 30 = 600$ When they understand the property of multiplication shown above, you can link this by showing you are multiplying the 2 and 3 then "annexing two zeros"- because the two zeros represent the 100 from the 10 x 10 part of the equation.

Example: $50 \times 60 = 3000$ This example shows the danger in saying "count the zeros and put that many on the answer"- because there are 3 zeros in the real answer!

- Area model of multiplication : CRITICAL to future understandings! It is essential that we show the area model PRIOR to shortening it to the "window pane" type model. Students need to understand that multiplication shows the area- not just the trick of multiplying by decomposing the numbers. The model isn't drawn to scale, but is approximate.



Notice that I've drawn the rectangle with the ROUGH approximation of size with the 20 being much longer than the 3

Distributive property of multiplication: decompose the factor(s) and multiply each addend separately then add the products.

Example: $23 \times 7 =$
 Decompose 23 into $(20 + 3)$
 Multiply each of the addends by 7
 $(20 \times 7) + (3 \times 7) =$
 $140 + 21 = 161$
 Connect this to the area model shown above.

Annex: appending the zeros (see example in right hand column)

“Window pane” model shows the same thing, but is not drawn in any kind of rough approximation of area. Make sure students understand WHY we can multiply this way before moving to the shorter step of using the window pane (equal size) boxes

Example

$$48 \times 6$$

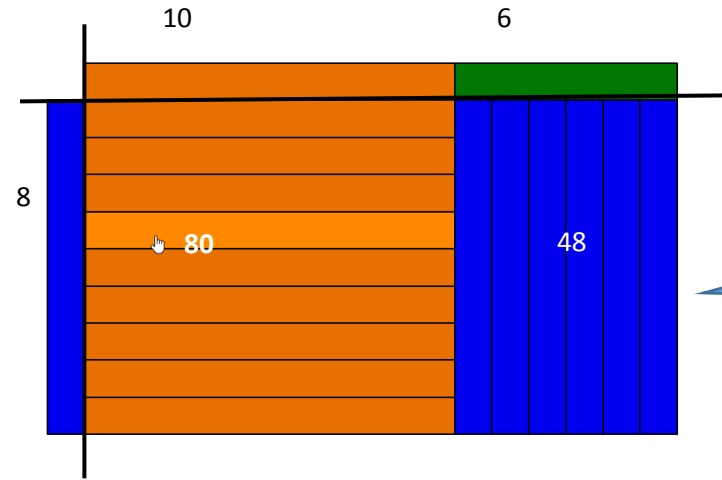
	40	8
6	240	48

This time I didn't worry about approximating the size and I made equal size boxes

Add the partial products together $240 + 48 = 288$

b) showing the area model for multiplication using Cuisenaire rods: this can help students see the distributive property and area model at work.

$$16 \times 8$$



Modeling with Cuisenaire rods: put the factors around the outside and then fill in the grid with the blocks. This allows students to visually see the “80” being made of eight 10 rods, and the “48” comprised of six 8 rods.

$$80 + 48 = 128$$

c) Once students really understand the area model, you can move into showing partial products in a more traditional format

$$\begin{array}{r} 23 \\ \times 7 \\ \hline 140 \\ + 21 \\ \hline 161 \end{array}$$

140 and 21 are the partial products that we add together to get the final product

Where does this lead?

Grade 5: 2 digit by 2 digit multiplication

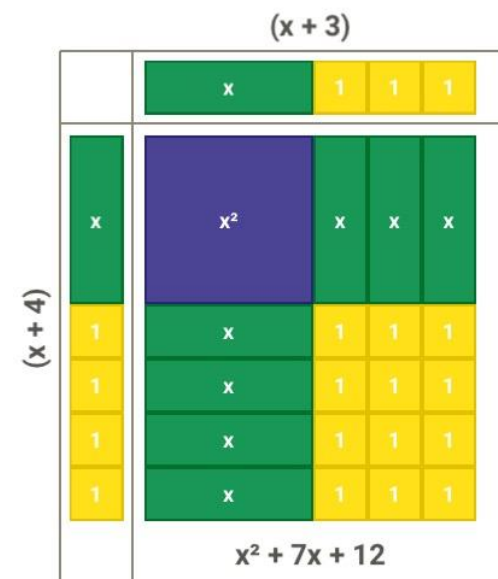
$$42 \times 23 = \quad (\text{again- model not drawn to scale, but approximated})$$

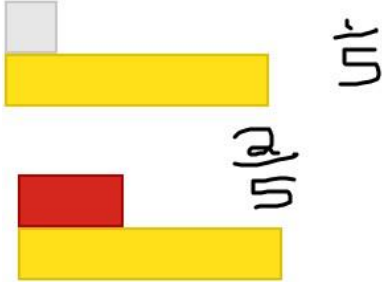
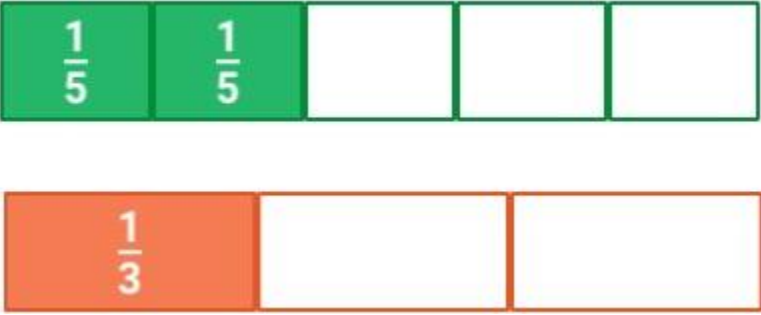
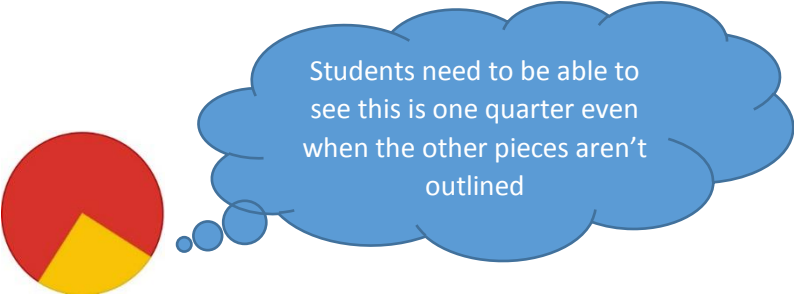
	40	2
20	800	40
3	120	6

Partial products $800 + 120 + 40 + 6 = 966$

Grade 10 example

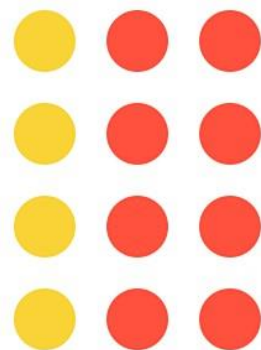
Multiplying binomials $(x + 3)(x + 4)$



<p>Curricular content</p> <p>Decimal fractions to hundredths</p> <p>Ordering and comparing fractions with common denominators</p> <p>Benchmarks of zero, half and whole</p>	<p>Examples and Strategies</p> <p>*Important to recognize that fractions are really the first time that students will have seen a situation where the value is not constant. The size of a fraction depends on the size of the whole. For example $\frac{1}{2}$ can be different sizes depending on the size of the whole. <u>Fractions show the relationship between the part and the whole, not the absolute size</u>* This is where Cuisenaire rods are really useful because it is easy to change the size of the whole (compared to using some other manipulatives where the whole is always the same size)</p> <p>The other key concept is that the pieces of the whole must be EQUAL.</p> <p>Three models for teaching about fractions:</p> <ol style="list-style-type: none"> 1) Length model (e.g. Cuisenaire rods) 2) Area model (e.g. Tiles or fraction circles) 3) Set model (e.g. counters) <p><u>Numerator</u>: counts <u>Denominator</u>: what is being counted</p>
<p>Language</p> <p>Numerator :how many we have</p> <p>Denominator: how many equal parts make up a whole</p> <p>Fractions are made up of equal size pieces (shares of a whole)</p> <p>**Important for naming decimals: 0.4 is said 4 tenths, not zero point four</p>	<p>Length model</p>  <p>Area model example</p>  

Decimals are a special type of fraction- they are always expressed as tenths, hundredths, thousandths etc. They can also be called decimal fractions. (Special type of fraction where the denominator is always 100)

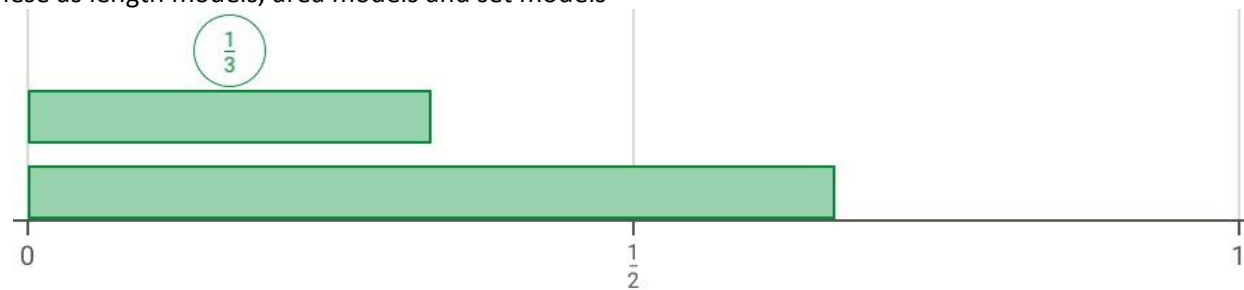
Set model



One third of the counters are yellow. Or 4/12 of the counters are yellow.

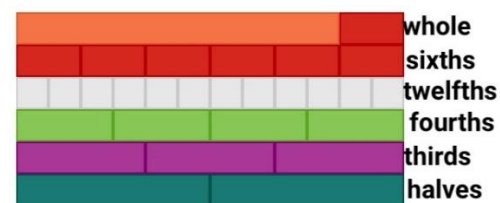
Benchmarks of zero, half and whole

Build these as length models, area models and set models



Where does this lead?

Equivalent fractions- really easy to see using Cuisenaire rods



From this diagram you can see many equivalent fractions (equivalent lengths). For example, you can see $\frac{3}{12} = \frac{1}{3}$ and $\frac{6}{12} = \frac{3}{6} = \frac{1}{2} = \frac{2}{4}$ etc

Linear functions: Grade 10: as a rate of change (slope of a line)

<p>Curricular content</p> <p>Decimals to hundredths</p>	<p>Examples and Strategies</p> <p>Start with a review of place value showing each place value is ten times larger/smaller than the column next to it</p> <div data-bbox="646 372 1072 614"> </div> <div data-bbox="1243 342 2315 624" style="background-color: #4a86e8; color: white; padding: 10px;"> <p>Each of the grey/white is one tenth (because it takes ten to make a whole orange). You can show that one tenth is written 0.1 or $\frac{1}{10}$. Decimals are fractions written with a multiple of ten as a denominator.</p> <p>If you continue putting tenths in the row, you have 11 tenths, or 1.1; 12 tenths or 1.2 etc</p> </div>
<p>Language</p> <p>decimals are a special type of fraction where the denominator is always 100</p> <p>Also called decimal fractions</p> <p>Naming decimals: instead of "point" we say "and"</p> <p>E.g. 1.62 is said One and sixty two hundredths</p>	<p>Model tenths using Cuisenaire rods as in the example above. Show that ten tenths is one whole.</p> <p>There are special manipulatives you can get that are built proportionally for decimals. (Decimal strips)</p> <p>If you don't have these, you can use the regular base ten blocks. Make sure students understand that you are calling the large cube "one".</p> <p>Name the cube "one whole"</p> <p>The flats are then tenths</p> <p>Rods are hundredths</p> <p>Unit cubes are now thousandths</p> <div data-bbox="646 1058 1072 1260"> </div> <p>Build numbers using this model. Practice naming. Example 1.04 is one and 4 hundredths</p> <p>1.205 is 1 and two hundred five thousandths</p> <p>An instructional strategy is to have students create these numbers by standing at the front of the room. Have one student be the "whole", another be the "tenth", another the "hundredths". Write a number on the board e.g. 1.4 Each student should then hold the appropriate amount of manipulatives. The person holding the hundredths should show they are holding nothing- this connects to being able to write zeroes on the end of the number without changing the quantity. Repeat with other numbers such as 2.36, 1.05 etc.</p> <p>Then move on to having students build these numbers at their desks.</p> <p>Be sure to compare decimals and be able to identify which of two or more numbers is larger. E.g. 1.6 and 1.06</p>

	<p>Where does this lead?</p> <p>All further operations with decimals.</p> <p>Decimals are used every year from now until grade 12 and beyond©</p>
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